Assignment 1 (CML100)

1. When lithium is irradiated with light, one finds a stopping potential of 1.83 V for $\lambda = 3000Å$ and 0.80 V for $\lambda = 4000Å$. From the data, calculate (a) Planck's constant, (b) threshold frequency, and (c) work function of Li.

2. Given $\hat{A} = d/dx$ and $\hat{E} = x^2$, show (a) $\hat{A}^2 f(x) \neq [\hat{A}f(x)]^2$ and (b) $\hat{A}\hat{E}f(x) \neq \hat{E}\hat{A}f(x)$ for arbitrary f(x).

3. Identify which of the following functions are eigenfunctions of the operators d/dx and d^2/dx^2: (a) e^{ikx} (b) cos kx (c) k (d) kx (e) e^{-ax^2}. Give the corresponding eigenvalue where appropriate
4. Find the eigenvalue in the following cases

$\hat{A}$ (operator)	f(x)	Eigenvalue
d^2/dx^2	$\cos \omega x$	
d/dt	$\exp(i\omega t)$	
$\frac{d^2}{dx^2} + 2\frac{d}{dx} + 3$	$\exp(\alpha x)$	
$\partial/\partial y$	$x^2 \exp(6y)$	

5. Show that $(\cos ax)(\cos by)(\cos cz)$ is an eigenfunction of the operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ which is known as the Laplacian Operator

6. Write out the operator for $\hat{A}^2$ for $\hat{A}$ =
   (a) $\frac{d^2}{dx^2}$ (b) $\frac{d}{dx} + x$ (c) $\frac{d^2}{dx^2} - 2x\frac{d}{dx} + 1$

7. Find the eigenfunctions and eigenvalues of the operator d/dx.
8. In algebra it can be easily shown that $(P + Q)(P - Q) = P^2 - Q^2$. What is the value of $(P + Q)(P - Q)$ if P and Q are operators? Under what conditions will this result be equal to $P^2 - Q^2$?

9. (a) Find $[d/dx, x]$, $[z^3, d/dz]$, $[d^2/dx^2, ax^2 + bx + c]$, and $[a, a^\#]$ where $a = (x + ip)/\sqrt{2}$ and $a^\# = (x - ip)/\sqrt{2}$. (b) Determine whether the operators SQR and SQRT commute.

10. Evaluate the commutator $[\hat{A}, \hat{B}]$, where $\hat{A}$ and $\hat{B}$ are given below

	$\hat{A}$	$\hat{B}$
(a)	$\frac{d}{dx} - x$	$\frac{d}{dx} + x$
(b)	$\frac{d^2}{dx^2} - x$	$\frac{d}{dx} + x^2$

15. Normalize the following wavefunctions to unity: ($\int_0^\infty x^n e^{-ax} dx = n!/a^{n+1}$)

(a) sin(nπx/L) for the range $0 \le x \le L$.
(b) c, a constant in the range $-L \le x \le L$.
(c) $\exp(-r/a_o)$ in 3-D.
(d) $x\exp(-r/2a_o)$ in 3-D.
(e) $(2-r/a_o)\exp(-r/2a_o)$ in 3-D.
(f) $r\sin\theta\cos\varphi\exp(-r/2a_o)$ in 3D

16. Two (unnormalized) excited state wavefunctions of the H-atom are (i) $\psi = \left(2 - \frac{r}{a_0}\right)\exp(-r/a_0)$ and (ii) $\psi = r\sin\theta\cos\phi\exp\left(-r/a_0\right)$. Normalize both functions to 1. Now show that these 2 functions are mutually orthogonal.

17. Which of the following candidates for wavefunctions are normalizable over the indicated intervals? Normalize those that can be normalized.

(a) $\exp\left(-x^2/2\right)$ (-∞, ∞) (b) e^x (0, ∞) (c) exp(iθ) (0, 2π) (d) xe^x (0, ∞) (e) $\exp\left[-\left(\frac{x^2+y^2}{2}\right)\right]$ (x,y: 0, ∞)

18. The wave function for a system can be written as $\psi(x) = \frac{1}{2}\phi_1(x) + \frac{1}{4}\phi_2(x) + \frac{3+i\sqrt{2}}{4}\phi_3(x)$ with $\phi_1(x)$, $\phi_2(x)$ and $\phi_3(x)$ being normalized eigenfunctions of the kinetic energy operator with eigenvalues E_1, $3E_1$ and $7E_1$ respectively. (a) Verify that ψ(*x*) is normalized. (b) What are the possible values of KE you will obtain in identically prepared systems. (c) What is the probability of measuring each of these eigenvalues? (d) What is the average value of kinetic energy that you would obtain from a large number of measurements.

[(c) 1/4, 1/16, 11/16 ; (d) $\langle E \rangle = 5.25E_1$ **]**